EE 330 Lecture 20

Bipolar Device Modeling

Fall 2025 Exam Schedule

Exam 1 Friday Sept 26

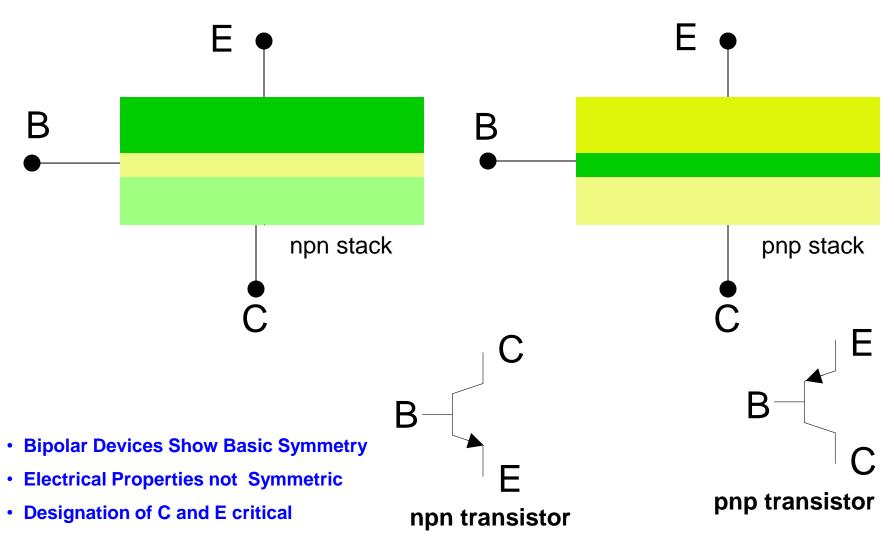
Exam 2 Friday October 24

Exam 3 Friday Nov 21

Final Exam Monday Dec 15 12:00 - 2:00

PM

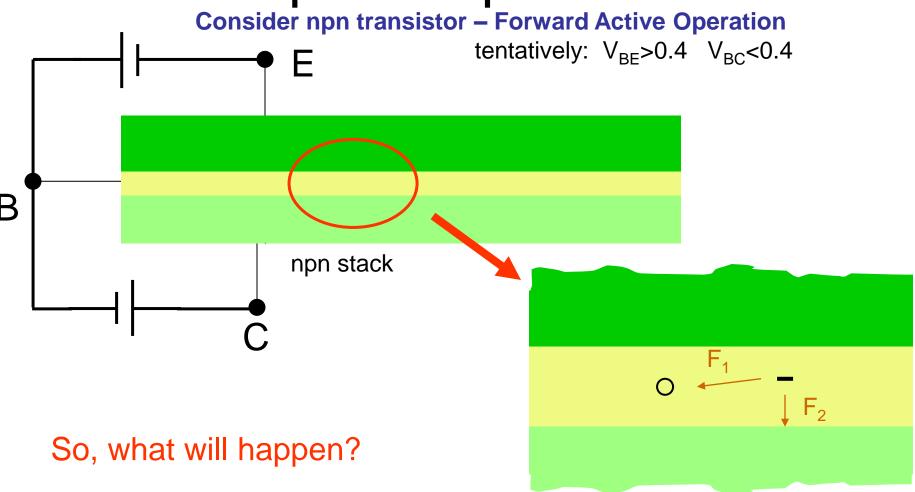
Bipolar Transistors



With proper doping and device sizing these form Bipolar Transistors

Review from Last Lecture

Bipolar Operation

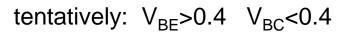


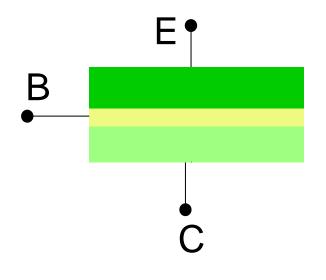
Some will recombine with holes and contribute to base current and some will be attracted across BC junction and contribute to collector

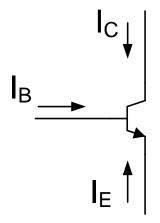
Size and thickness of base region and relative doping levels will play key role in percent of minority carriers injected into base contributing to collector current

Bipolar Operation

Consider npn transistor – Forward Active Operation







$$I_{C} + I_{B} = -I_{E}$$

$$I_{C} = -\alpha I_{E}$$

$$\beta = \frac{1 - \alpha}{1 - \alpha}$$

$$\beta = \frac{\alpha}{1 - \alpha}$$

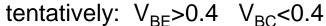
$$I_{C} = \beta I_{B}$$

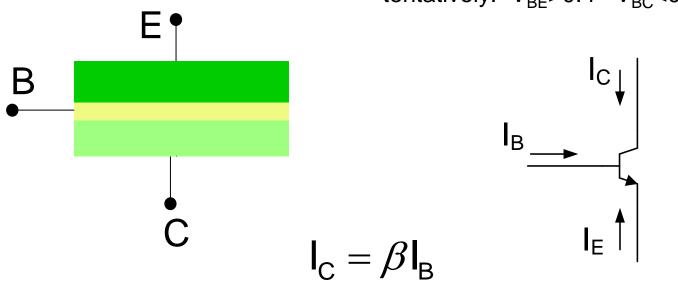
β is typically very large

often 50<β<999

Review from Last Lecture Bipolar Operation

Consider npn transistor – Forward Active Operation





β is typically very large

Bipolar transistor can be thought of as current amplifier with a large current gain

In contrast, MOS transistor is inherently a tramsconductance amplifier

Current flow in base is governed by the diode equation

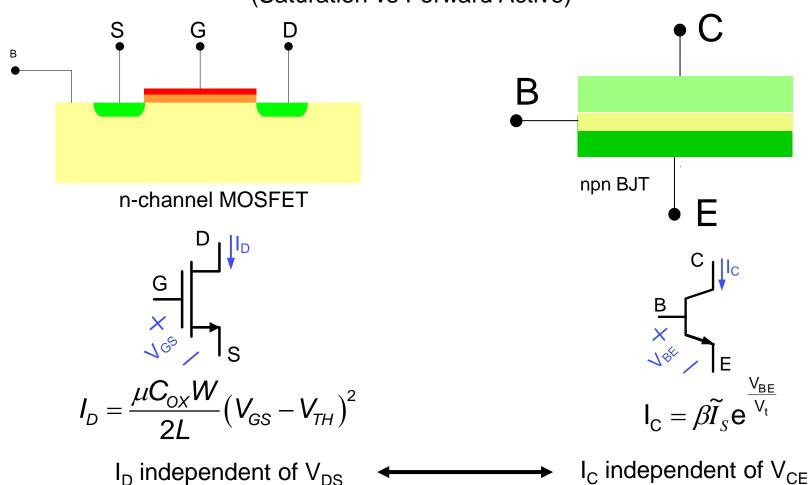
Collector current thus varies exponentially with V_{BE}

$$I_{B} = \widetilde{I}_{S} e^{\frac{BL}{V_{t}}}$$

$$I_{C} = \beta \widetilde{I}_{S} e^{\frac{V_{BE}}{V_{t}}}$$

Preliminary Comparison of MOSFET and BJT

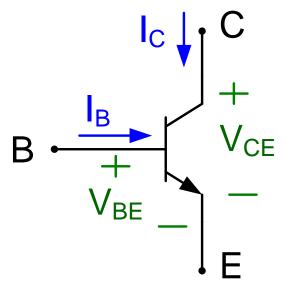
(Saturation vs Forward Active)



- The BJT I/O relationship is exponential in contrast to square-law for MOSFET
- Provides a very large "gain" for the BJT (assuming input is voltage and output is current)
- This property is very useful for many applications

Bipolar Models

Simple dc Model



Following convention, pick I_C and I_B as dependent variables and V_{BE} and V_{CE} as independent variables

Simple dc model

npn transistor - Forward Active Operation

$$I_{B} = \frac{J_{S}A_{E}}{\beta}e^{\frac{V_{BE}}{V_{t}}}$$

$$I_{C} = J_{S}A_{E}e^{\frac{V_{BE}}{V_{t}}}$$

$$V_{t} = \frac{kT}{\alpha}$$

As with the diode, the parameter J_S is highly temperature dependent

$$\mathbf{J}_{s} = \mathbf{J}_{sx} \left[\mathbf{T}^{m} \mathbf{e}^{\frac{-\mathbf{V}_{so}}{\mathbf{V}_{t}}} \right]$$

Typical values for parameters: $J_{SX}=20\text{mA/}\mu^2$, $V_{G0}=1.17\text{V}$, m=2.3

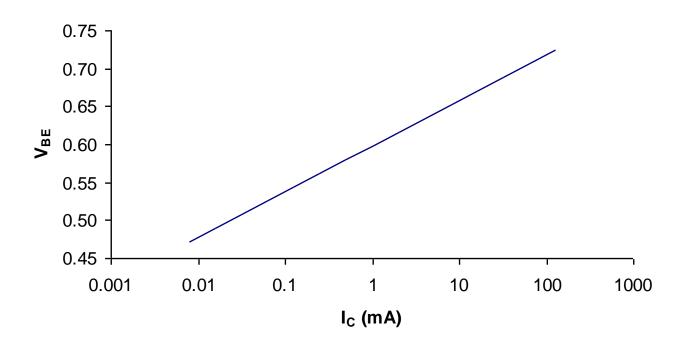
The parameter β is also somewhat temperature dependent but much weaker temperature dependence than J_{SX} .

Transfer Characteristics

npn transistor – Forward Active Operation

$$J_S = .25fA/u^2$$

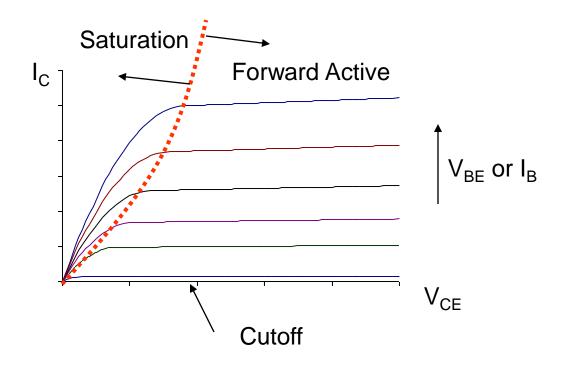
 $A_F = 400u^2$



V_{BE} close to 0.6V for a four decade change in I_C around 1mA

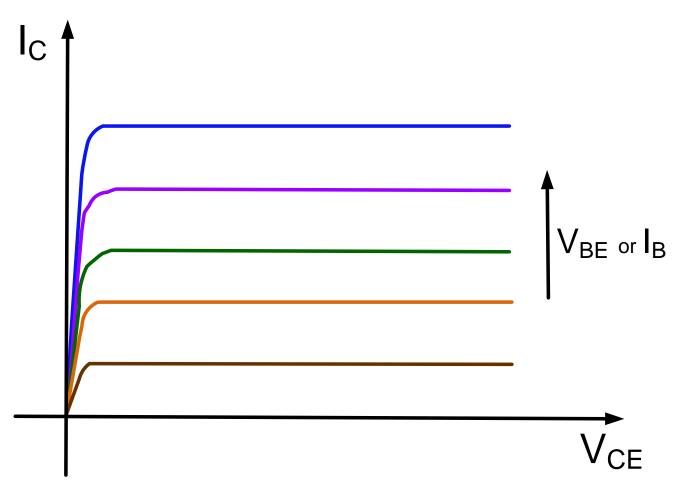
Simple dc model

Typical Output Characteristics



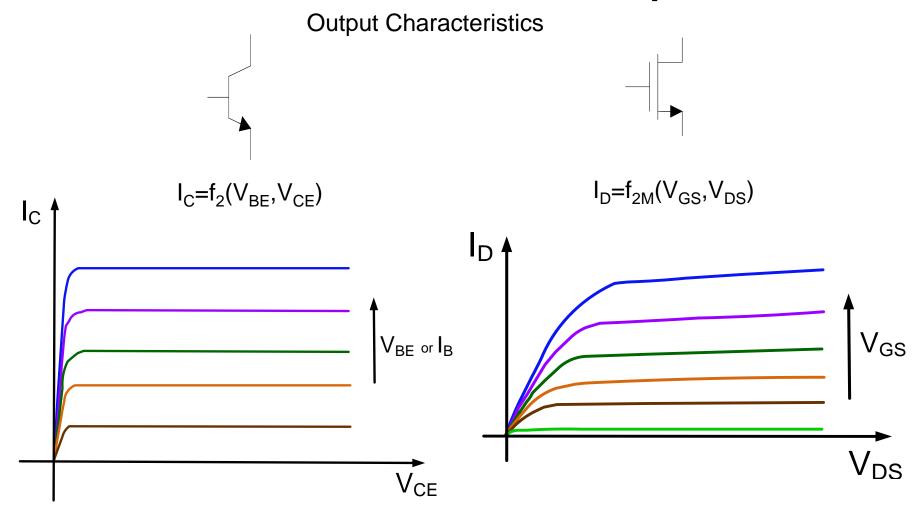
Forward Active region of BJT is analogous to Saturation region of MOSFET Saturation region of BJT is analogous to Triode region of MOSFET

Better Model of Output Characteristics



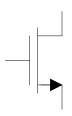
With scaled V_{CE} axis, transition in saturation very steep

BJT and MOSFET Comparison



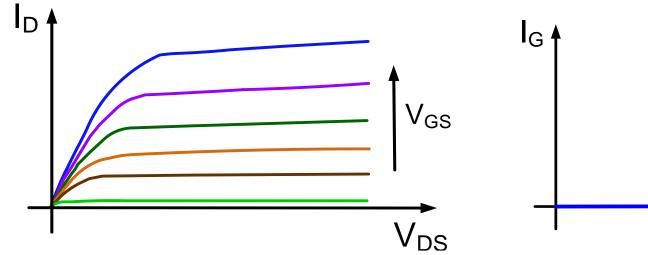
- Same general characteristics
- Spacings a bit different (Exponetial vs square law)
- Slope steeper for small V_{CE} compared to small V_{DS}

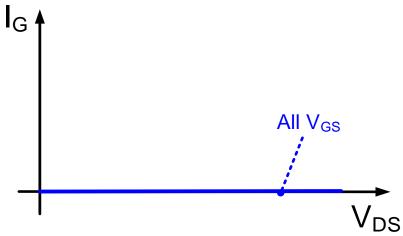
Recall MOSFET Operation



Output characteristics

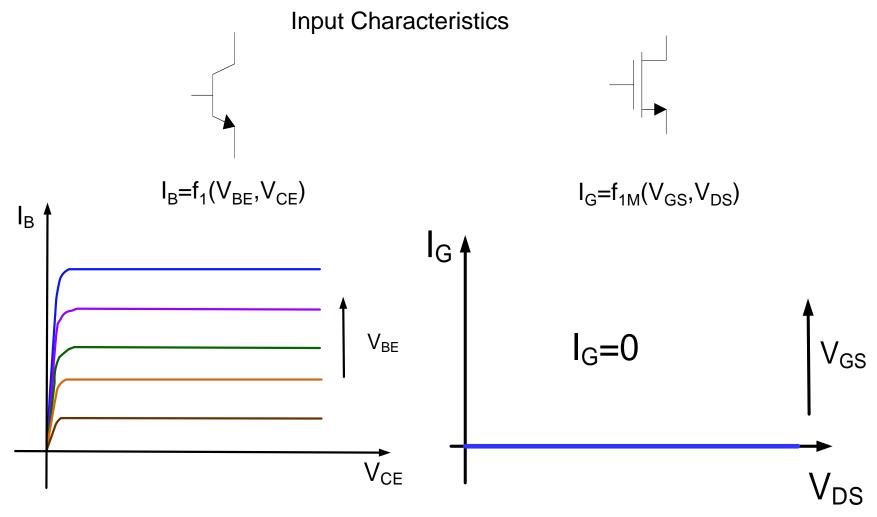
Input characteristics





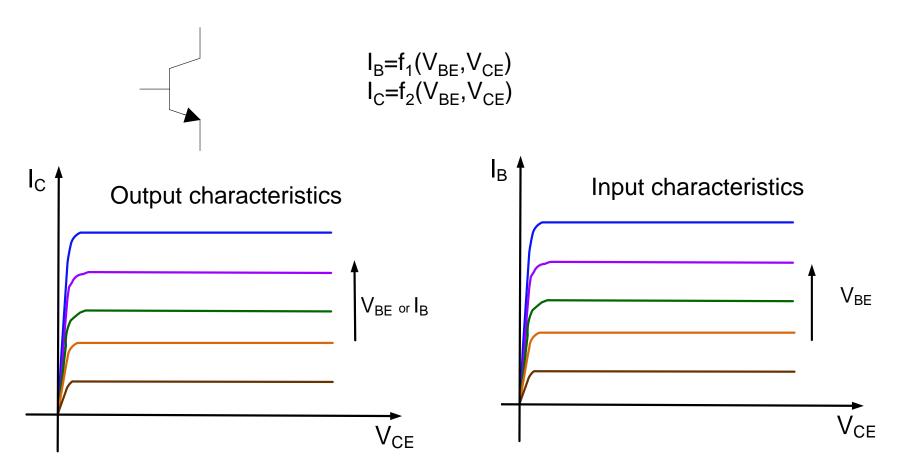
or equivalently: $I_{G}=0$

BJT and MOSFET Comparison



Did not need to graphically show input characteristics for MOS transistors since I_G=0

BJT Model

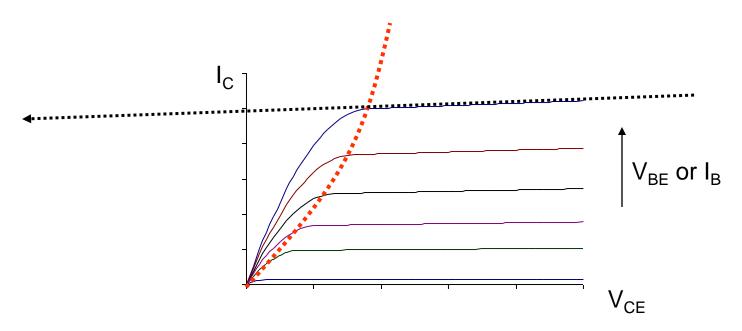


Require two graphical representations (or analytical expressions) though vertical axis scales different by factor of β

Since $I_B=f(V_{BE})$, can use independent (V_{BE}) or dependent (I_B) variable for 2-D visualization of 3-dimensional I_C function

Improved simple dc model

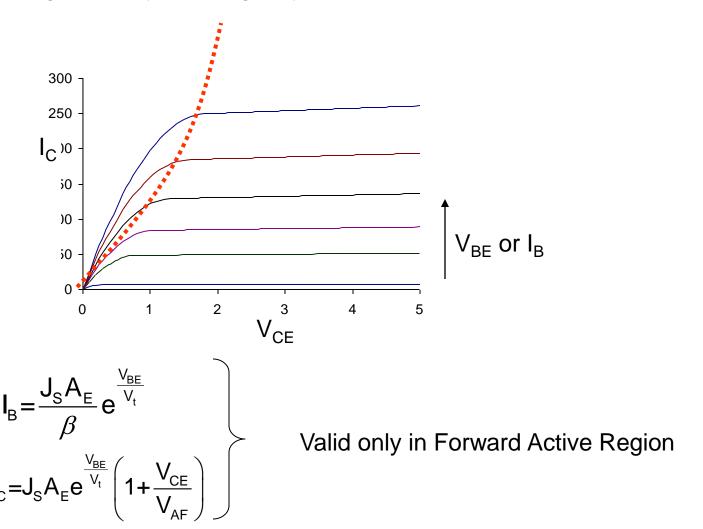
Typical Output Characteristics



- Projections of these tangential lines all intercept the $-V_{CE}$ axis at the same place and this is termed the Early voltage, V_{AF} (actually $-V_{AF}$ is intercept)
- Typical values of V_{AF} are in the 100V to 200V range
- Can multiply expression for I_C in Forward Active Region by term $\left(1+\frac{V_{CE}}{V_{AF}}\right)$ to account for slope

Improved simple dc model

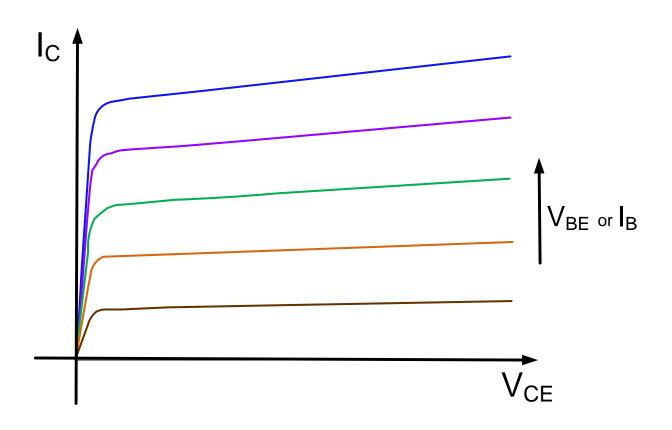
(graphically showing only output characteristics)



Need models in saturation and cutoff regions

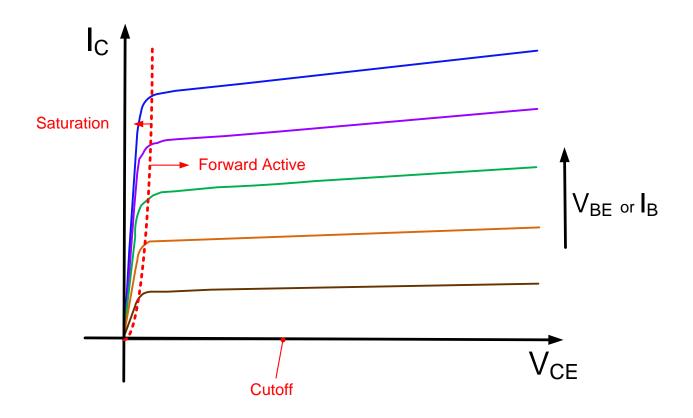
Improved simple BJT dc model

Typical Output Characteristics



Improved simple BJT dc model

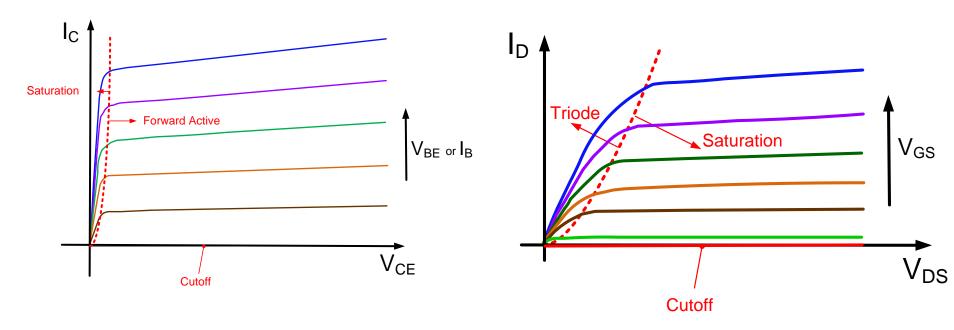
Typical Output Characteristics



Need analytical models in saturation and cutoff regions

Improved simple BJT dc model

Typical Output Characteristics

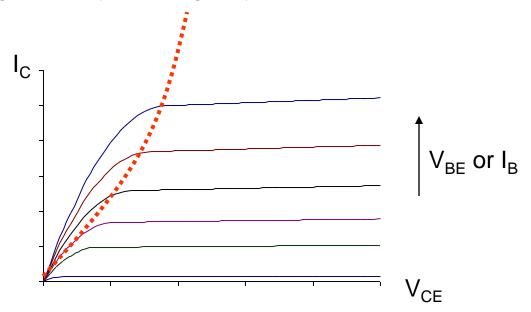


Recall:

Forward Active region of BJT is analogous to Saturation region of MOSFET Saturation region of BJT is analogous to Triode region of MOSFET

Improved dc model

(graphically showing only output characteristics)



$$V_t = \frac{kT}{q}$$

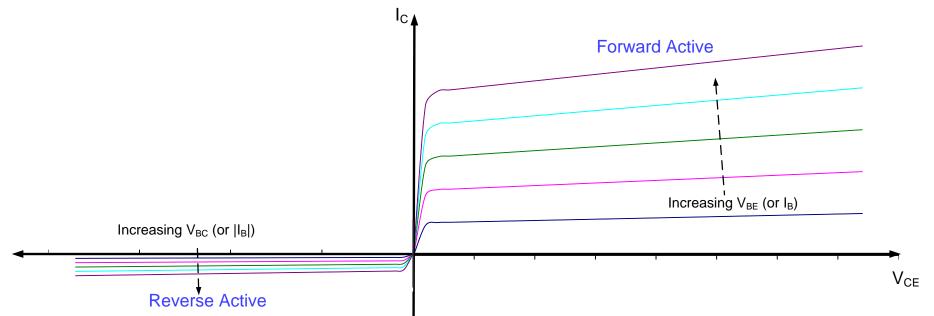
$$I_{E} = -\frac{J_{S}A_{E}}{\alpha_{F}} \left(e^{\frac{V_{BE}}{V_{t}}} - 1 \right) + J_{S}A_{E} \left(e^{\frac{V_{BC}}{V_{t}}} - 1 \right)$$

$$I_{C} = J_{S}A_{E} \left(e^{\frac{V_{BE}}{V_{t}}} - 1 \right) - \frac{J_{S}A_{E}}{\alpha_{R}} \left(e^{\frac{V_{BC}}{V_{t}}} - 1 \right)$$

- Valid in All regions of operation
- V_{AF} effects can be added
- Not mathematically easy to work with
- Note dependent variables changes {I_E,I_C}
- Termed Ebers-Moll model
- Reduces to previous model in FA region
- Little use in Reverse Active Region

Improved dc model

(graphically showing only output characteristics)



$$V_t = \frac{kT}{q}$$

$$I_{E} = -\frac{J_{S}A_{E}}{\alpha_{F}} \left(e^{\frac{V_{BE}}{V_{t}}} - 1 \right) + J_{S}A_{E} \left(e^{\frac{V_{BC}}{V_{t}}} - 1 \right)$$

$$I_{C} = J_{S}A_{E} \left(e^{\frac{V_{BE}}{V_{t}}} - 1 \right) - \frac{J_{S}A_{E}}{\alpha_{R}} \left(e^{\frac{V_{BC}}{V_{t}}} - 1 \right)$$

- Model using I_E and I_C as dependent variables
- Valid in All regions of operation
- V_{AF} effects can be added
- Not mathematically easy to work with
- Note dependent variables changes
- Termed Ebers-Moll model
- Reduces to previous model in FA region
- Little use in Reverse Active Region

Ebers-Moll model

$$I_{E} = -\frac{J_{S}A_{E}}{\alpha_{F}} \left(e^{\frac{V_{BE}}{V_{t}}} - 1 \right) + J_{S}A_{E} \left(e^{\frac{V_{BC}}{V_{t}}} - 1 \right)$$

$$I_{C} = J_{S}A_{E} \left(e^{\frac{V_{BE}}{V_{t}}} - 1 \right) - \frac{J_{S}A_{E}}{\alpha_{R}} \left(e^{\frac{V_{BC}}{V_{t}}} - 1 \right)$$

Process Parameters: $\{J_S, \alpha_F, \alpha_R\}$ $V_t = \frac{kT}{q}$

Design Parameters: {A_E}

 α_F is the parameter α discussed earlier α_R is termed the "reverse α "

$$\beta_{F} = \frac{\alpha_{F}}{1 - \alpha_{F}} \qquad \beta_{R} = \frac{\alpha_{R}}{1 - \alpha_{R}} \qquad \Longrightarrow \qquad \alpha_{F} = \frac{\beta_{F}}{1 + \beta_{F}} \qquad \alpha_{R} = \frac{\beta_{R}}{1 + \beta_{R}}$$

Typical values for process parameters:

$$J_S \sim 10^{-16} A/\mu^2$$
 $\beta_F \sim 100$, $\beta_R \sim 0.4$

Can substitute for α_F and α_R in Ebers-Moll model

Ebers-Moll model

$$I_{E} = -\frac{J_{S}A_{E}}{\alpha_{F}} \left(e^{\frac{V_{BE}}{V_{t}}} - 1 \right) + J_{S}A_{E} \left(e^{\frac{V_{BC}}{V_{t}}} - 1 \right)$$

$$I_{C} = J_{S}A_{E} \left(e^{\frac{V_{BE}}{V_{t}}} - 1 \right) - \frac{J_{S}A_{E}}{\alpha_{R}} \left(e^{\frac{V_{BC}}{V_{t}}} - 1 \right)$$

With typical values for process parameters in forward active region $(V_{BE}\sim0.6V V_{BC}\sim-3 V_{t}\sim26mV)$ and if $A_{E}=100\mu^{2}$

$$I_{C} = \frac{10^{-14} \left(1.05 \times 10^{10} - 1\right) - 3.6 \times 10^{-14} \left(7.7 \times 10^{-14} - 1\right)}{\text{Completely dominant}}$$

Makes no sense to keep anything other than $I_C = J_S A_E e^{\frac{V_{BE}}{V_t}}$ in forward active region

Ebers-Moll model

Ebes-Moll model
$$I_{E} = -\frac{J_{S}A_{E}}{\alpha_{F}} \left(e^{\frac{V_{BE}}{V_{t}}} - 1 \right) + J_{S}A_{E} \left(e^{\frac{V_{BC}}{V_{t}}} - 1 \right)$$

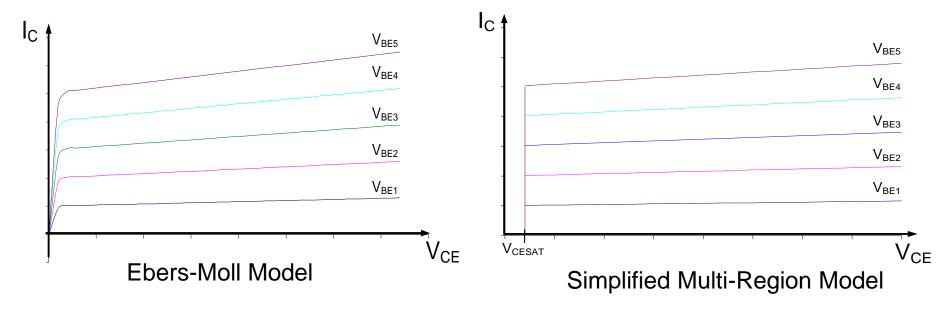
$$V_{t} = \frac{kT}{q} \qquad I_{C} = J_{S}A_{E} \left(e^{\frac{V_{BE}}{V_{t}}} - 1 \right) - \frac{J_{S}A_{E}}{\alpha_{P}} \left(e^{\frac{V_{BC}}{V_{t}}} - 1 \right)$$

Alternate equivalent expressions for dependent variables $\{I_C, I_B\}$ defined earlier for Ebers-Moll equations in terms of independent variables $\{V_{BE}, V_{CE}\}$ after dropping the "-1" terms

$$\begin{split} I_{c} &= J_{s}A_{E}e^{\frac{V_{BE}}{V_{t}}}\left(1 - \left[\frac{1 + \beta_{R}}{\beta_{R}}\right]e^{\frac{-V_{CE}}{V_{t}}}\right) \\ I_{B} &= J_{s}A_{E}e^{\frac{V_{BE}}{V_{t}}}\left(\frac{1}{\beta_{E}} - \frac{1}{\beta_{R}}e^{\frac{-V_{CE}}{V_{t}}}\right) \end{split}$$

No more useful than previous equation but in form consistent with notation Introduced earlier

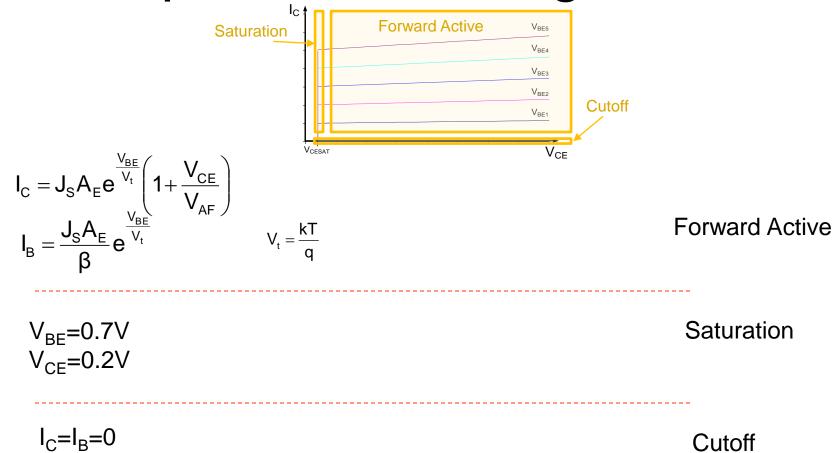
(graphically showing only output characteristics)



- Observe V_{CE} around 0.2V when saturated
- V_{BF} around 0.6V when saturated
- In most applications, exact V_{CE} and V_{BE} voltage in saturation not critical

Simplified model in saturation:

$$V_{BE}=0.7V$$
 Saturation $V_{CE}=0.2V$



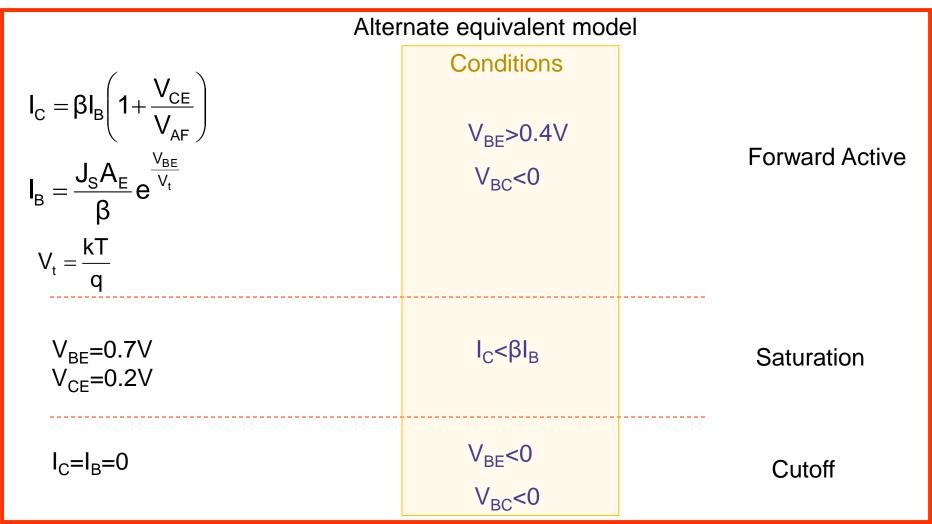
- This is a piecewise model suitable for analytical calculations
- Can easily extend to reverse active mode but of little use
- Still need conditions for operating in the 3 regions !!

"Forward" Regions : $\beta = \beta_F$

	Conditions	
$I_{C} = J_{S}A_{E}e^{\frac{V_{BE}}{V_{t}}}\left(1 + \frac{V_{CE}}{V_{AF}}\right)$	$V_{BE}>0.4V$ $V_{BC}<0$	
$I_{B} = \frac{J_{S}A_{E}}{\beta}e^{\frac{V_{BE}}{V_{t}}}$		Forward Active
V _{BE} =0.7V V _{CE} =0.2V	I _C <βI _B	Saturation
I _C =I _B =0	V _{BE} <0 V _{BC} <0	Cutoff

Process Parameters: $\{J_S, \beta, V_{AF}\}\ \{V_{BESAT}, V_{CESAT}\}\ V_t = \frac{kT}{q}$ Design Parameters: $\{A_E\}$

- Process parameters highly process dependent
- J_S highly temperature dependent as well, β modestly temperature dependent
- This model is dependent only upon emitter area, independent of base and collector area!
- Currents scale linearly with A_E and not dependent upon shape of emitter
- V_{BESAT} and V_{CESAT} often not considered process parameters since change little with process
- A small portion of the operating region is missed with this model but seldom operate in the missing region

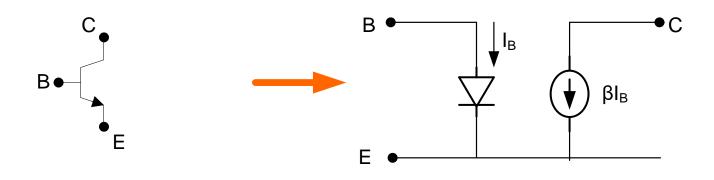


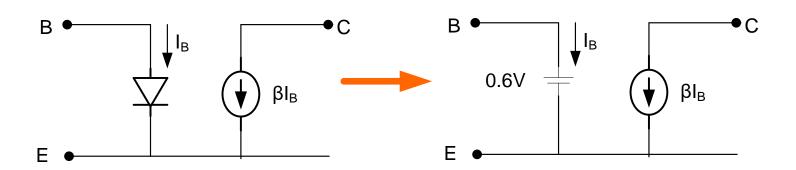
A small portion of the operating region is missed with this model but seldom operate in the missing region

Further Simplified Multi-Region dc Model

(by neglecting V_{AF})

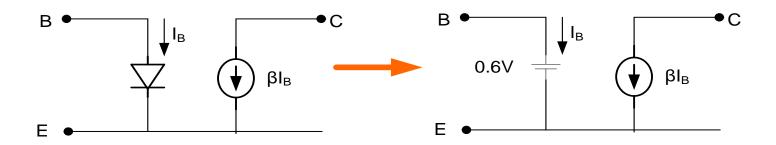
Forward Active





Adequate when it makes little difference whether $V_{BE}=0.6V$ or $V_{BE}=0.7V$

Forward Active

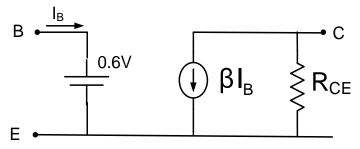


Mathematically

$$V_{BE}=0.6V$$
 $I_{C}=\beta I_{B}$

Or, if want to show slope in I_C-V_{CE} characteristics

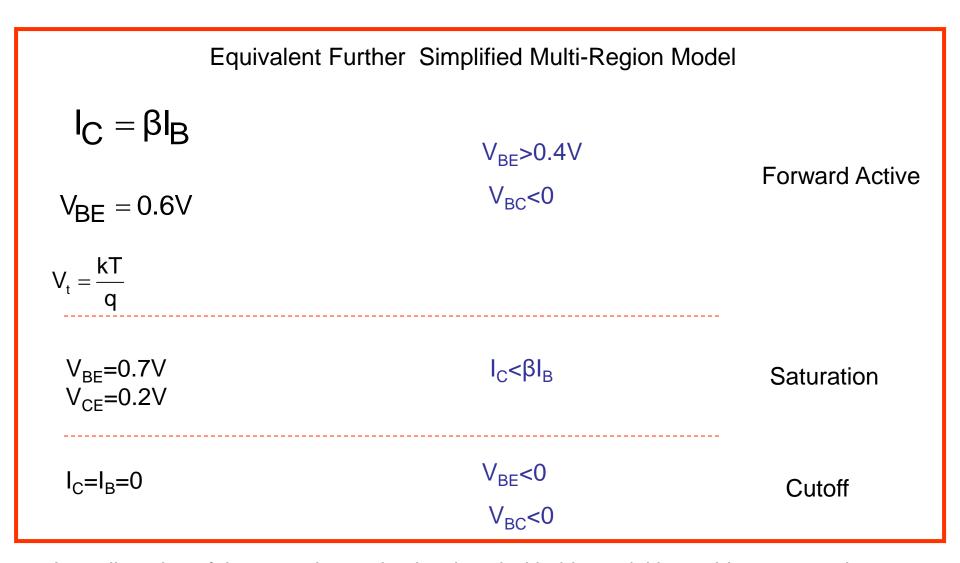
$$V_{BE}$$
=0.6 V_{CE}



$$R_{CE} = \frac{V_{AF}}{\beta I_{BO}}$$

R_{CE} highly nonlinear

Further Simplified Multi-Region dc Model



A small portion of the operating region is missed with this model but seldom operate in the missing region

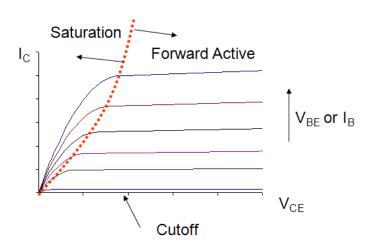
Conditions for Regions of Operation in Multi-Region Model

$$V_{BE}>0.4V$$
 $V_{BC}<0$
Forward Active

 $I_{C}<\beta I_{B}$
Saturation

 $V_{BE}<0$
 $V_{BC}<0$
Cutoff

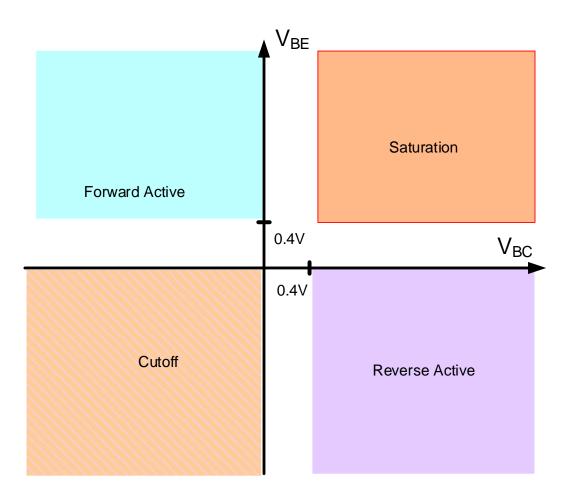
Note: One condition is on dependent variables!



Observe that in saturation, $I_C < \beta I_B$

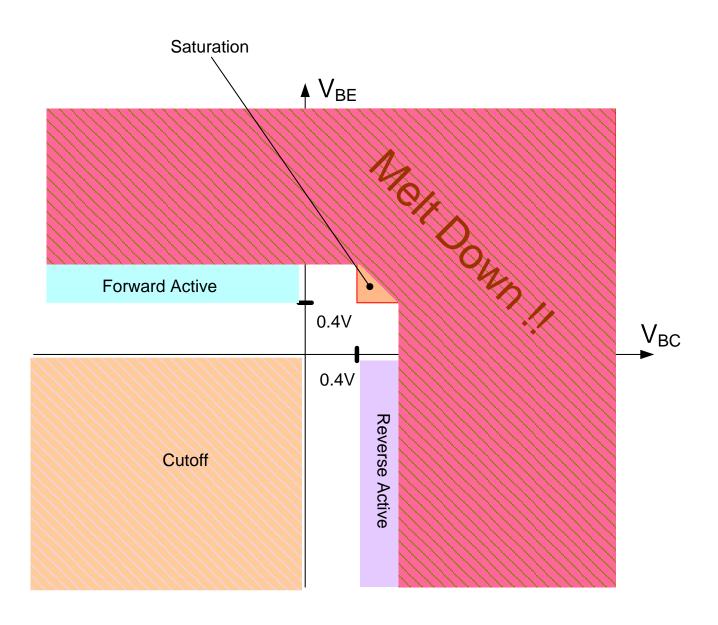
Can't condition on independent variables in saturation because they are fixed in the model

Regions of Operation in Independent Parameter Domain used In multi-region models

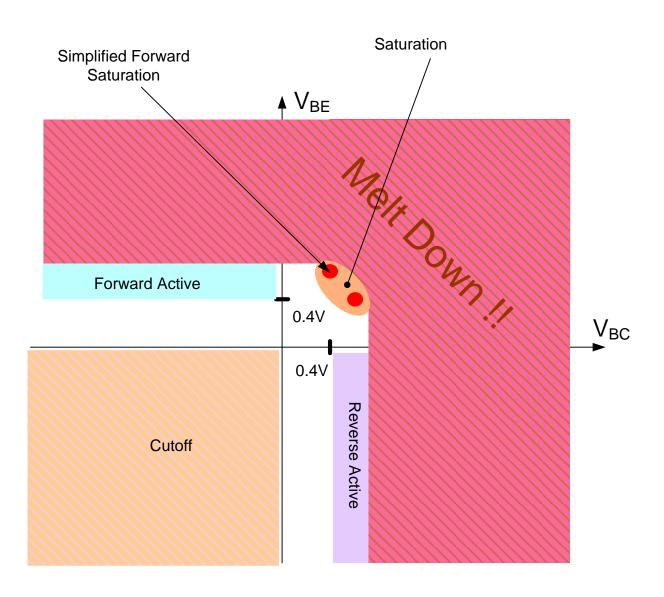


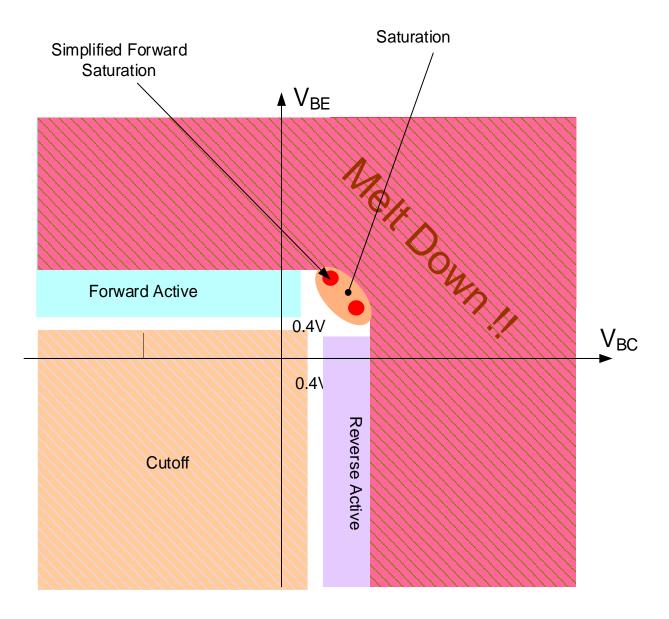
- Seldom operate in regions excluded in this picture
- Limited use in Reverse Active Mode

Excessive Power Dissipation if either junction has large forward bias



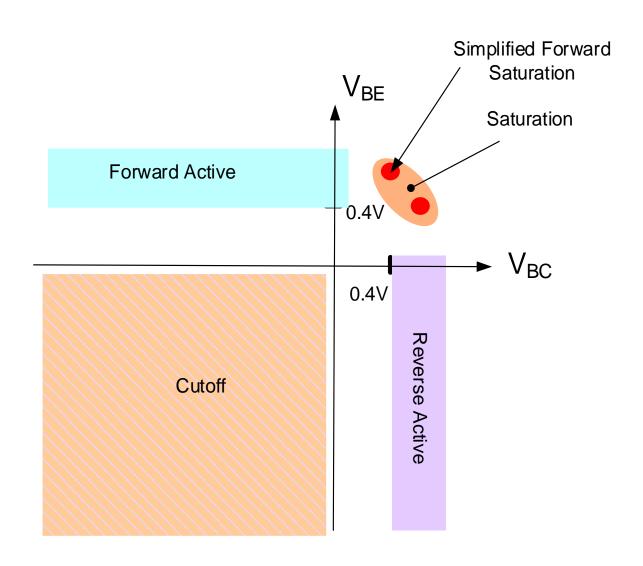
Safe regions of operation



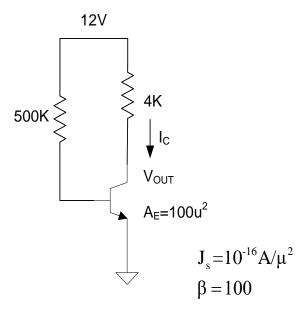


Actually cutoff, forward active, and reverse active regions can be extended modestly as shown and multi-region models still are quite good

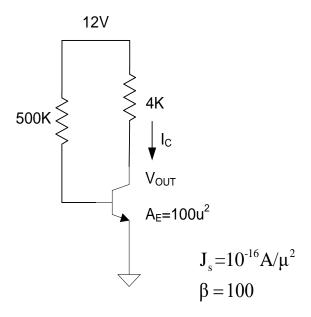
Sufficient regions of operation for most applications



Example: Determine I_C and V_{OUT}

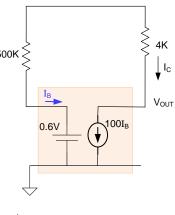


Example: Determine I_C and V_{OUT}



Solution:

- 1. Guess Forward Active Region (and model)
- 2. Solve Circuit with Guess
- 3. Verify model (if necessary)



$$I_B = \frac{(12 - 0.6)}{500K}$$

$$I_C = \beta I_B = 100 \frac{(12 - 0.6)}{500K} = 2.28mA$$

$$V_{OUT} = 12 - I_C \bullet 4K = 2.88V$$

4. Verify FA Region

$$V_{BE} = 0.6V > 0.4V$$
 $V_{BE} > 0.4V$ and $V_{BC} < 0$
 $V_{BC} = 0.6V - 2.88V = -2.28V < 0$

Verify Passes so solution is valid

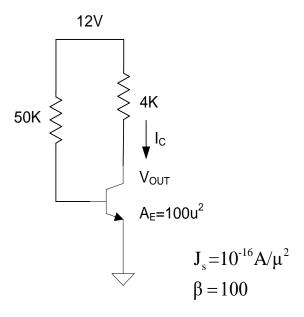
$$I_C = 2.28mA$$
$$V_{OUT} = 2.88V$$

5. Verify model (if necessary)

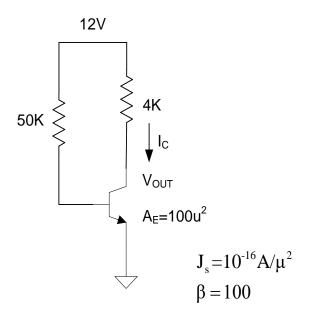
Solve again with $V_{BE}=0.7V$ Will show $V_{OUT}=2.96V$ so difference is small

Note solution independent of J_S and A_E

Example: Determine I_C and V_{OUT},

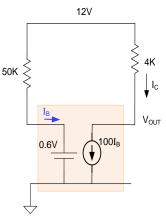


Example: Determine I_C and V_{OUT} .



Solution:

- 1. Guess Forward Active Region
- 2. Solve Circuit with Guess
- 3. Verify model (if necessary)



$$I_{B} = \frac{(12-0.6)}{50K}$$

$$I_{C} = \beta I_{B} = 100 \frac{(12-0.6)}{50K} = 22.8mA$$

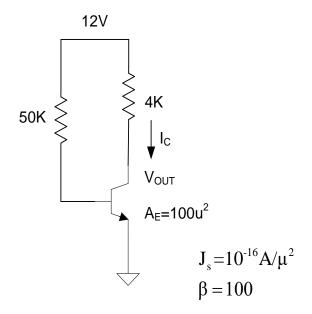
$$V_{OUT} = 12 - I_{C} \cdot 4K = -79.2V$$

4. Verify FA Region $V_{BE}>0.4V$ and $V_{BC}<0$

$$\begin{split} V_{\rm BE} &= 0.6V \ > 0.4V \\ V_{\rm BC} &= 0.6V - -79.2V = +79.8V > 0 \end{split}$$

Verify Fails so solution is not valid

Example: Determine I_C and V_{OUT}



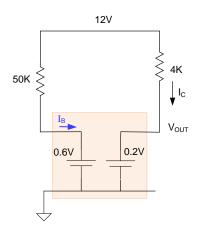
Solution:

- 5. Guess Saturation
- 6. Solve Circuit with Guess
- 7. Verify model (if necessary)

$$I_{B} = \frac{(12-0.6)}{50K} = 228\mu A$$

$$I_{C} = \frac{(12-0.2)}{4K} = 2.95mA$$

$$V_{OUT} = 0.2V$$



 $I_{C} < \beta I_{R}$

8. Verify SAT Region

$$\beta I_B = 100 \cdot 228 \mu A = 22.8 mA$$

 $I_C = 2.95 mA$
 $I_C = 2.95 mA < \beta I_B = 22.8 mA$

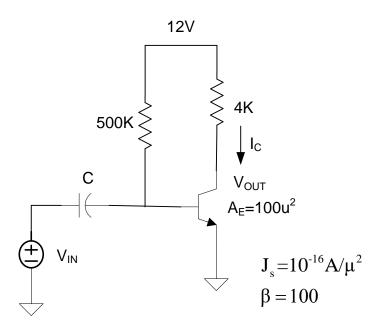
Verify SAT Passes so solution is valid

$$I_C = 2.95mA \qquad V_{OUT} = 0.2V$$

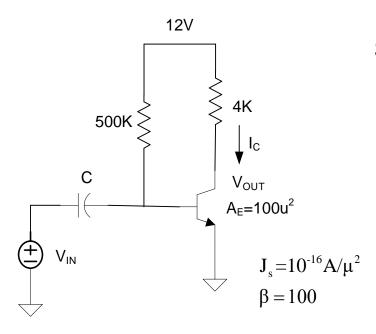
9. Verify model (if necessary)

(use V_{BE} =0.7V, no change in output)

Example: Determine I_C and V_{OUT} . Assume C is large and V_{IN} is very small.



Example: Determine I_C and V_{OUT} . Assume C is large and V_{IN} is very small.



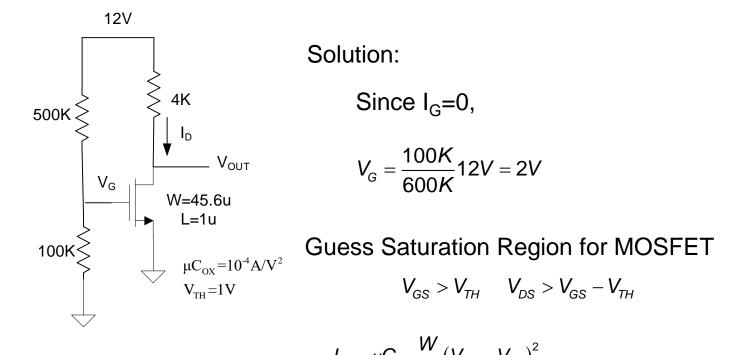
Solution:

Assume V_{IN}=0, then no current flows through C so circuit is identical to circuit of previous-previous example so

$$I_C = 2.28mA \qquad V_{OUT} = 2.88V$$

Note: If C is large and V_{IN} is small sinusoidal signal of sufficiently high frequency, the voltage across C will not change the input so V_{IN} is from an ac viewpoint coupled directly to base. In this case, the circuit will amplify V_{IN} and the gain will be very large due to the exponential relationship between I_{C} and V_{BF} .

Example: Determine I_D and V_{OUT}



Solution:

$$V_{\rm G} = \frac{100K}{600K} 12V = 2V$$

$$V_{\rm GS} > V_{\rm TH}$$
 $V_{\rm DS} > V_{\rm GS} - V_{\rm TH}$

$$I_D = \mu C_{OX} \frac{W}{2I} (V_{GS} - V_{TH})^2$$

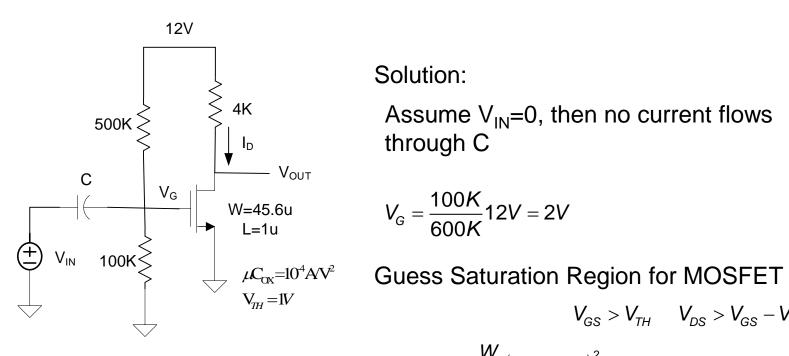
$$I_D = 10^{-4} \frac{45.6}{2} (2-1)^2 = 2.28 \text{ mA}$$

$$V_{OUT} = 2.88V$$

Verify saturation 2V > 1V 2.88V > 2V - 1V

Note: solution dependent upon W,L,V_{TH}, and uC_{ox}

Example: Determine I_D and V_{OUT} . Assume C is large and V_{IN} is very small.



Solution:

Assume $V_{IN}=0$, then no current flows through C

$$V_{\rm G} = \frac{100K}{600K} 12V = 2V$$

$$V_{\rm GS} > V_{\rm TH}$$
 $V_{\rm DS} > V_{\rm GS} - V_{\rm TH}$

$$I_{D} = \mu C_{OX} \frac{W}{2L} (V_{GS} - V_{TH})^{2}$$

$$I_D = 10^{-4} \frac{45.6}{2} (2-1)^2 = 2.28 mA$$

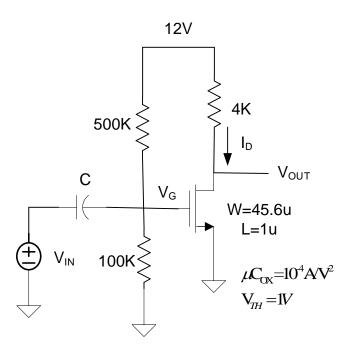
$$V_{OUT} = 2.88V$$

Verify saturation 2V > 1V 2.88V > 2V - 1V

This circuit has the same current and same V_{OUT} as the previous circuit

solution dependent upon W,L,V_{TH}, and uC_{ox}

Example: Determine I_D and V_{OUT} . Assume C is large and V_{IN} is very small.



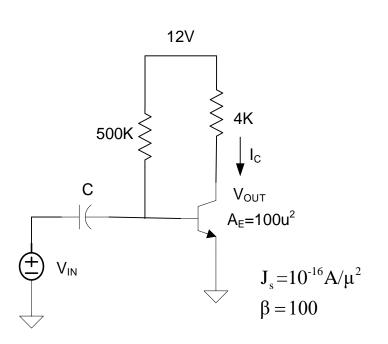
Solution:

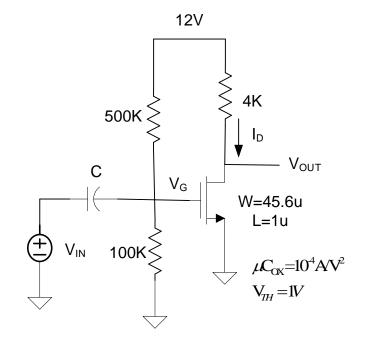
Assume V_{IN}=0, then no current flows through C so circuit is identical to circuit of previous-previous example so

$$I_C = 2.28mA \qquad V_{OUT} = 2.88V$$

Note: If C is large and V_{IN} is small sinusoidal signal of sufficiently high frequency, the voltage across C will not change so V_{IN} is from an ac viewpoint coupled directly to gate. In this case, the circuit will amplify V_{IN} and the gain will be large due to the square-law relationship between I_D and V_{GS} .

Comparison





$$I_{c} = I_{D} = 2.28 \text{mA}$$
 $V_{OUT} = 2.88 \text{V}$

$$V_{OUT}=2.88V$$

- Both circuits can serve as amplifiers
- Architectures very similar
- MOS circuit dependent upon model parameters, BJT independent of model parameters
- Will be shown later that the bipolar circuit has larger gain because exponential vs square law relationship



Stay Safe and Stay Healthy!

End of Lecture 20